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Ballistic Trajectory Analysis

This study was conceived as a method to test a previous theory regarding predicting a shooter's location from multiple gunshot strikes into the sheet metal of a patrol car.

In 2010, I testified in a tragic capital punishment case involving a deputy shot at seventeen times with an AR15 from approximately 80 feet away. One round pierced the deputy's chest plate and he died shortly after. In analyzing the available physical evidence to locate the shooter's specific location, I used Mike Haag's previously derived "error" or variance rate of +/- 5 degrees for each individual trajectory. When visualized in the 3D Working Model, this variance value appears as a 3 dimensional cone surrounding the derived trajectory. The cone's base grows in size as one moves farther away from the impact point for each round, developing into a fairly large area at the distances we were analyzing. This large area made determining whether the shooter was inside or outside the adjacent residence difficult, as the variance extend to a diameter of approximately 16 feet for each individual shot.



I noted, however, that if I analyzed the shots as a group as opposed to individually, a different picture began to emerge. When visualized in 3D as a group, there was an area where all the trajectories overlapped – an area within which all the data was being matched, and therefore an area that contained locations for the shooter that were consistent with all of the physical evidence.





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At the time of the trial, I calculated and testified that the statistically most likely location for the shooter was at the geometric center of this overlap area. This location had the lowest mean squared error when compared to the individual derived trajectories. (See more at: http://tinyurl.com/kk8ec4w)

The current study provided an opportunity to test this theory and potentially provide a method to increase the accuracy of our prediction of the shooter's location over that afforded by using the trajectories individually.

GENERAL SETUP:

The study was comprised of eight different test conditions – two caliber/weapon types (.45 caliber handgun vs .223 caliber AR15 semi-automatic rifle), two targets (dual-ply drywall with 4 inches of airspace between them vs. car doors) and two angles of impact (90 degrees vs 45 degrees).

Gun Type	Target Type	Target Angle
.223 AR15 Rifle	Drywall	90°
.223 AR15 Rifle	Drywall	45°
.45 Handgun	Drywall	90°
.45 Handgun	Drywall	45°
.223 AR15 Rifle	Car Door	90°
.223 AR15 Rifle	Car Door	45°
.45 Handgun	Car Door	90°
.45 Handgun	Car Door	45°

Each test was conducted at a distance of 90 feet to avoid drop in the relatively slow handgun rounds.

The weapons were located in a fixed position using a Ransom Rest to lock down the AR15 and to support the .45 handgun. The targets for each weapon type were laid out side by side and fired in series before switching them out for the next weapon type. The entire scene, including the weapon location in the Ransom Rest and the targets were documented through a 3D laser scan using our Leica[®] Geosystems ScanStation C10.

Each target/angle setup was fired at twelve times from each weapon. Each round fired was clocked using radar to determine the speed of the round at a distance of approximately 1 foot after leaving the weapon's muzzle.



After the twelve rounds were fired, each impact location was fitted with a custom made trajectory rod. The rods used in the study were powder coated a flat primer gray to increase the resolution of the scan data and limit the artifacts often seen with traditional trajectory rods. After the set of twelve impact sites were fit with the trajectory rods, each rod was documented for azimuth and elevation with the ScanStation.

Once all the test data had been captured with the ScanStation, it was imported into Autodesk's 3D Studio MAX software for 3-dimensional analysis. Each of the eight test conditions were analyzed separately in the computer. The trajectory rods that were scanned in each impact site were traced back in a straight line to the plane of the weapon's muzzle. One of many benefits of doing this work in the computer and using the 3D Working Model, is that if the computer is good at anything, drawing straight lines is certainly one of them. Given the lack of drop expected in the rounds over a distance of 30 yards, a straight line is the best model of the bullet's true trajectory. Performing this work in the field would add unnecessary error to the underlying analyses as projecting truly straight lines would prove all but impossible.

For each test condition, the location of the straight line traceback (predicted shooter's position) where it crossed the plane of the muzzle was visualized and compared to the known location of the weapon. This comparison resulted in a 2-axis Cartesian grid, with the weapon's known location located at the grid's origin and each individual predicted location (the point where the traceback intersected the muzzle's plane at 30 yards or 90 feet) shown on the grid. This method provided an intuitive and functional data set for visualizing and measuring both the spread of the data and the accuracy.







.45 Handgun - Drywall - 90° - 1 Foot Grid

As shown in the above diagram, the known location of the weapon is located at the origin of the grid, with 1 foot intervals for the gridlines.

The blue boxes illustrate the predicted location of the weapon for each of the twelve rounds fired, based upon the traceback's position at the plane of the weapon's muzzle at 90 feet. The red "star" is the geometric center of the group of predicted locations (tracebacks).

Analysis of the data:

Using the grid method illustrated above, each test condition was reviewed and important data extracted regarding the relative accuracy and spread of the predicted locations. Basic data compiled included sample size, maximum/minimum errors from known accuracy of predicted locations - individual, maximum/minimum and average spread of error (standard deviation of error/precision).



Each trajectory was fitted with the currently accepted +/- 5 degree cone of uncertainty and the overlap of these cones was visualized on the 2D grid. In the example below, the known weapon location is at the grids origin, the predicted locations are shown as blue boxes, the +/- 5 degree cones are shown in green and their overlap area is shaded in light blue:



.45 Handgun - 45° Incidence Angle Drywall -Default 5° Cones - 1 Foot Grid

What we discovered was that, although using the geometric center of the overlap area did indeed increase the accuracy over the individual trajectories, it was not the most accurate indicator to fall out of the data. As it turned out, the geometric center of the group of predicted locations was the most accurate indicator, reducing the error in predicted locations by as much as 20 times over using the average error across the dataset. In the above example, the red star illustrates the predicted location using the geometric center of the individual tracebacks – an error of less than 0.5 feet over 90 feet! The following graphics illustrate this effect for all 8 test conditions:



The following graphics illustrate the results of tracebacks (predicted shooter location) with both weapons shot through drywall:



.45 Handgun - Drywall - 90° - 1 foot grid



The known location of the weapon is located at the origin of the grid. The blue boxes illustrate the predicted location (traceback) of the weapon. The red "star" is the geometric center of the group of predicted locations (tracebacks).



The following graphics illustrate the results of tracebacks (predicted shooter location) with both weapons shot through car door:



.45 Handgun - Car Door- 90° - 1 foot grid

The known location of the weapon is located at the origin of the grid. The blue boxes illustrate the predicted location (traceback) of the weapon. The red "star" is the geometric center of the group of predicted locations (tracebacks).



^{.45} Handgun - Car Door - 45° - 1 foot grid

As is apparent from a brief review of the data plots, the predicted location based upon this geometric average greatly reduces the error over both the maximum error and the average error of the individual traceback locations. The following illustrates this effect numerically:

Gun Type	Target Type	Target Angle	Average Error of Traceback at 90 Feet	
.223 AR15 Rifle	Drywall	90°	0.90°	1.41 ft.
.223 AR15 Rifle	Drywall	45°	0.75°	1.18 ft.
.45 Handgun	Drywall	90°	0.82°	1.30 ft.
.45 Handgun	Drywall	45°	0.90°	1.41 ft.
.223 AR15 Rifle	Car Door	90°	1.02°	1.60 ft.
.223 AR15 Rifle	Car Door	45°	1.71°	2.69 ft.
.45 Handgun	Car Door	90°	2.51°	3.95 ft.
.45 Handgun	Car Door	45°	3.18°	5.00 ft.

			Geometric Center of Individual Tracebacks at 90 Feet	
Gun Type	Target Type	Target Angle	Total Error Degrees	Total Error Feet
.223 AR15 Rifle	Drywall	90°	0.45°	0.706 ft.
.223 AR15 Rifle	Drywall	45°	0.36°	0.570 ft.
.45 Handgun	Drywall	90°	0.04°	0.057 ft.
.45 Handgun	Drywall	45°	0.29°	0.455 ft.
.223 AR15 Rifle	Car Door	90°	0.50°	0.784 ft.
.223 AR15 Rifle	Car Door	45°	0.34°	0.538 ft.
.45 Handgun	Car Door	90°	0.94°	1.484 ft.
.45 Handgun	Car Door	45°	2.12°	3.338 ft.

Traceback = Predicted Shooter Position



Confidence Intervals/Validity per test Condition

Previous work performed by others explored the concept of confidence intervals for the accuracy of any given trajectory traceback. Based upon this work, the value of +/- 5 degrees has been suggested and adopted by many who work in the field. The previous work on this issue has focused on a statistical approach based upon standard deviation of the error in large sets of predicted locations. We were unsatisfied with this approach for reasons of both mathematical validity, as well as having a single value of +/- 5 degrees for any and all measurement conditions.

A new approach was developed and applied to the data in this study. As the concept of "cones of uncertainty" presented by previous authors was well accepted and intuitive, we chose to work with the existing framework of an error cone. However, the calculation of the error cones we used was graphical as opposed to statistical.

The data in our study provided direct comparison for each round fired between the predicted location and the actual (known) location of the weapon, as previously shown. When viewed on a Cartesian grid, the predicted locations are readily compared to the known (which lies at the origin of each Cartesian grid) and the direction and amount of error for each traceback is readily apparent. This method of illustrating the resultant predicted locations as opposed to analyzing the angular components provides a more intuitive and functionally useful illustration of the ultimate goal – determining the shooters location, not the angles of the individual shots themselves.

Although none of the rounds tested exhibited zero error – none of them exactly predicted the true shooters location – the degree of and error pattern for each condition provides a visual reference for the relative accuracy. In determining what type of confidence or size of error cone would best be applied to each test condition we chose to look at the minimum size of error cone that would still result in every cone containing the known shooter's location. This approach has the benefit of being visual and intuitive – if we are after a high degree of certainty in our predictions, our error cones should always contain the known location. The resulting cones would take into account both the average error and the spread of each set of predictions, as would be expected in an analysis of validity and confidence. The tighter the spread and closer to the known location, the smaller the level of uncertainty and therefore the smaller the error cone. In addition, this method ensures that our ultimate prediction of a shooters location takes into account all of the available evidence, an important requisite when presenting this data in trial.

The following graphics illustrate this method. Each test condition is shown twice – first with the previously accepted +/- 5 degree cones and then again with the resultant cones scaled to the smallest size where all the cones contain the known shooter's location. Note that in the .45 handgun, car door, 45° impact angle condition, the cone size needed to encompass the known location was *larger* than 5 degrees; and in the .45 handgun, car door, 90° impact angle condition, the scaled cone size need to encompass all the data *was* 5 degrees.





Default 5° Cones - 1 Foot Grid

.223 AR15 Rifle - 45° Incidence Angle - Drywall -Minimum Cone Radius = 1.7° - 1 Foot Grid





.45 Handgun - 90° Incidence Angle - Drywall -Default 5° Degree Cones - 1 Foot Grid



.45 Handgun - 90° Incidence Angle - Drywall -Minimum Cone Radius = 2.0° - 1 Foot Grid



.45 Handgun - 45° Incidence Angle - Drywall -Default 5° Cones - 1 Foot Grid



.45 Handgun - 45° Incidence Angle - Drywall -Minimum Cone Radius = 1.80° - 1 Foot Grid





.223 AR15 Rifle - 90° Incidence Angle - Car Door -Default 5° Cones - 1 Foot Grid



.223 AR15 Rifle - 90° Incidence Angle - Car Door -Minimum Cone Radius = 2.6 - 1 Foot Grid



.223 AR15 Rifle - 45° Incidence Angle -Car Door -Default 5° Cones - 1 Foot Grid

.223 AR15 Rifle - 45° Incidence Angle- Car Door -Minimum Cone Radius = 2.9° - 1 Foot Grid

= Cone Overlap





.45 Handgun - 90° Incidence Angle- Car Door -Default 5° Cones - 1 Foot Grid



.45 Handgun - 90° Incidence Angle - Car Door -Minimum Cone Radius = 5.0° - 1 Foot Grid * Minimum Radius same as Default 5.0°



.45 Handgun - 45° Incidence Angle - Car Door -Default 5° Cones - 1 Foot Grid *Default 5° cone overlap did not encompass known location



.45 Handgun - 45° Incidence Angle - Car Door -Minimum Cone Radius = 5.4° - 1 Foot Grid *Default 5° cone overlap did not encompass known location



When analyzed numerically in this manner, the error cones, or "cones of uncertainty" are as follows for each test condition:

Gun Type	Target Type	Target Angle	Cone Radius in Feet	Cone Radius in Degrees
.223 AR15 Rifle	Drywall	90°	2.60 ft.	1.7°
.223 AR15 Rifle	Drywall	45°	2.60 ft.	1.7°
.45 Handgun	Drywall	90°	3.18 ft.	2.0°
.45 Handgun	Drywall	45°	2.85 ft.	1.8°
.223 AR15 Rifle	Car Door	90°	4.16 ft.	2.6°
.223 AR15 Rifle	Car Door	45°	4.63 ft.	2.9°
.45 Handgun	Car Door	90°	7.80 ft.	5.0°
.45 Handgun	Car Door	45°	8.50 ft.	5.4°

Relative Contribution to Variance of Test Variables

When looking at the raw data, the largest contribution to the variance in predicted location accuracy comes from the target material. Of the eight test conditions, the data for the car doors displayed the lowest accuracy, occupying all four of the lowest rankings; the data for the drywall occupied all four of the highest accuracy. This affect is also illustrated by the difference in the average errors – the comparison between the four car door conditions and the four drywall conditions nets the largest difference – 2.1° for the car door data vs 0.84° for the drywall data, a difference of 1.26°.

The variable that contributed the 2nd most to the predicted variance was the weapon/caliber. The data for the AR15 rifle firing .223 caliber rounds had an average error of 1.09° versus the handgun firing .45 caliber rounds with 1.85° across test conditions. The variable that contributed the least to the variance was the angle of incidence – the 1.63° average error for the .45° condition vs the 1.31° of error for the 90° condition results in a difference of only 0.32°.



Gun Type	Target Type	Target Angle	Average Error in Degrees
.223 AR15 Rifle	Drywall	45°	0.75°
.45 Handgun	Drywall	90°	0.82°
.223 AR15 Rifle	Drywall	90°	0.90°
.45 Handgun	Drywall	45°	0.90°
.223 AR15 Rifle	Car Door	90°	1.02°
.223 AR15 Rifle	Car Door	45°	1.71°
.45 Handgun	Car Door	90°	2.51°
.45 Handgun	Car Door	45°	3.18°

Test Conditions Rankings in Ascending Order:

Comparison by Weapon Type:

Gun Type	Average Error in Degrees
.223 AR15 Rifle	1.09°
.45 Handgun	1.85°
Difference	0.76°

Comparison by Angle of Incidence:

Angle of Incidence	Average Error in Degrees
90°	1.31°
45°	1.63°
Difference	0.32°

Comparison by Target Type:

Target	Average Error in Degrees
Drywall	0.84°
Car Door	2.10°
Difference	1.26°



Errors at 90 Feet - Per Test Condition:

.223 AR15 Rifle through Drywall at 90°	.223 AR15 Rifle through Drywall at 45°
Sample Size (N)=12	Sample Size (N)=12
Maximum Error = 2.48 ft./1.58°	Maximum Error = 2.51 ft./1.60 °
Minimum Error = 0.21 ft./0.13°	Minimum Error = 0.31 ft./0.20°
Average Error = 1.41 ft./0.90°	Average Error = 1.17ft./0.75°
Standard Deviation Errors = 0.72 ft./0.46°	Standard Deviation Errors = 0.66 ft./0.42°
Error from Arithmetic Average = 0.06 ft.	Error from Arithmetic Average = 0.54 ft.
Error from Cone Overlap Center = 0.51 ft.	Error from Cone Overlap Center = 1.62 ft.
.45 Handgun through Drywall at 90°	.45 Handgun through Drywall at 45°
Sample Size (N)=10	Sample Size (N)=12
Maximum Error = 3.08 ft./1.96 °	Maximum Error = 2.69 ft./1.71°
Minimum Error = 0.15 ft./0.10 °	Minimum Error = 0.28 ft/0.18 °
Average Error = 1.30 ft./0.82°	Average Error = 1.41 ft./0.90 °
Standard Deviation Errors = 0.85 ft./0.54°	Standard Deviation Errors = 0.79 ft./0.51°
Error from Arithmetic Average = 0.71 ft.	Error from Arithmetic Average = 0.45 ft.
Error from Cone Overlap Center = 1.45 ft.	Error from Cone Overlap Center = 0.57 ft.
.223 AR15 Rifle through Car Door at 90°	.223 AR15 Rifle through Car Door at 45°
Sample Size (N)=12	Sample Size (N)=10
Maximum Error = 4.03 ft./2.56°	Maximum Error = 4.51 ft./2.87 °
Minimum Error = 0.40 ft./0.25°	Minimum Error = 1.30 ft./0.83°
Average Error = 1.60ft./1.02°	Average Error = 2.68ft./1.71°
Standard Deviation Errors = 1.15 ft./0.73°	Standard Deviation Errors = 1.06 ft./0.67°
Error from Arithmetic Average = 0.784 ft.	Error from Arithmetic Average = 0.54 ft.
Error from Cone Overlap Center = 1.87 ft.	Error from Cone Overlap Center = 1.62 ft.
.45 Handgun through Car Door at 90°	<u>.45 Handgun through Car Door at 45°</u>
Sample Size (N)=12	Sample Size (N)= 9
Maximum Error = 7.69 ft./4.89 °	Maximum Error = 8.4 ft./5.3°
Minimum Error = 1.12 ft./0.71°	Minimum Error = 0.19 ft./0.12°
Average Error = 3.94ft./2.51°	Average Error = 5.0 ft./3.18°
Standard Deviation Errors = 1.78 ft./1.08 °	Standard Deviation Errors = 2.59 ft./1.64°
Error from Arithmetic Average = 1.48 ft.	Error from Arithmetic Average = 3.34 ft.
Error from Cone Overlap Center = 2.16 ft.	Error from Cone Overlap Center = 2.38 ft.



Speed of Fired Rounds, for each Test Condition (feet per second)

.223 AR15 Rifle					
Round Number	45° - Drywall	45° - Car Door	90° - Drywall	90° - Car Door	
1	2917	2879	2949	2949	
2	2911	2911	2917	2909	
3	2871	2917	2898	2917	
4	2861	2867	2892	2917	
5	2892	2898	2879	2749	
6	2830	2911	2930	2930	
7	2867	2904	2930	2949	
8	2911	2911	2923	2873	
9	2892	2390	2892	2911	
10	2855	2911	2936	2898	
11	2867	2930	2911	2923	
12	2886	2949	2898	2867	
Average Speed	2880	2865	2913	2899	

.45 Handgun					
Round Number	45° - Drywall	45° - Car Door	90° - Drywall	90° - Car Door	
1	802	816	804	829	
2	799	825	803	820	
3	838	826	839	824	
4	827	819	828	825	
5	836	840	836	825	
6	832	832	822	819	
7	828	834	819	812	
8	822	828	817	812	
9	847	817	817	835	
10	823	826	819	835	
11	819	825	809	806	
12	827	829	826	806	
Average Speed	825	826	820	821	



Conclusion

This study was conceived in 2010 after we faced the challenge of determining the shooters location from seventeen shots fired from an AR-15 semi-automatic rifle. Upon examination of the evidence in that case and subsequent cases two things became clear: the previously derived +/- 5 degree cones of uncertainty were prohibitively conservative; and that there was functionally useful data to be gleaned from assessing the fired rounds as a set, as opposed to individually.

From visualizing the data in 3D as was first done in our 2010 case work, it was apparent that in cases with multiple rounds fired from a single location, the individual tracebacks described a s tatistical "cloud" surrounding the actual shooters location. What was needed was a method to utilize the multiple predicted locations in a way that took into account both their spread and their relative accuracy. The concept of the overlap area of the +/- 5 degree cones was appealing both visually and by the virtue of this location matching all the available evidence – a critical component of validity when testifying to the results.

After our tests were completed, analysis of the results revealed the following:

- 1. The intended goal of the study was to determine whether the geometric center of the overlap of the +/- 5 degree cones accurately predicted the shooters location. In all 8 test scenarios this datum did predict the shooters location with increased accuracy over using the average error of the set of traceback predictions. However it was determined that a more accurate prediction was provided by using the geometric center of the set of traceback predictions, essentially ignoring the cones and their common overlap area. It should be noted that this more accurate datum always lies within the cones overlap error however it was not located at the overlaps geometric center.
- 2. For all but one of our test conditions (pistol firing .45 caliber rounds through a car door at a 45 degree angle of incidence) the previously derived +/-5 degree cones were larger than necessary to fully account for the spread and absolute values of the errors in predicting the shooter's location.
- 3. Given that one of our condition resulted in an error cone of 5.4 degrees and a second resulted in error cones of 5.0 degrees, the previously derived +/- 5 degree cones are valid for a broad value that applies over a wide range of conditions.
- 4. The test conditions that resulted in large error were both from relatively large and slow .45 caliber rounds fired into car doors. In these test conditions, many of the rounds lacked sufficient velocity to make a secondary hole in the back of the target, thereby limiting the accuracy of the resultant traceback. If encountered in live casework, it would be necessary to hold the rods resting in the single bullet hole against the "pinch or wipe" point to increase predictive accuracy. In the scenarios using the .45 caliber handgun fired into the car door at a 45 degree angle, a few of the rounds lacked sufficient momentum to make a single hole in the front side of the target, causing the data to be discarded as there was no hole into which a trajectory round could be inserted.



- 5. The single largest factor in resultant prediction accuracy was the target material. The ability of each caliber to fully perforate both layers of the drywall provided two points of data between which a straight line traceback could be derived. The .45 caliber handgun had particular difficulty in perforating the car door.
- 6. The weapon/caliber variable was the second most important variable affecting prediction accuracy.
- 7. The angle of incidence contributed the least to the prediction accuracy. As long as two holes were available, the accuracy in predicting the shooters location was very high regardless of weapon type.
- 8. The use of the ScanStation, the custom-made trajectory rods and analysis of the data in the computer using the 3D Working Model provided for very high prediction accuracy across most condition (all conditions where two holes were available.) At a distance of 90 feet from the physical evidence in the form of bullet holes, this method was able to predict shooters location to within an average of 1.6 feet in 5 of the test conditions and within 5 feet in the worst condition, using the average error.
- Using the set of the data and analyzing the geometric center of the individual predictions of shooters location provided even greater accuracy – up to 20x better than using the average error. At 90 feet from the physical evidence, this datum predicted the shooters location to within 3.3 feet in the worst case scenario and as accurate as **0.05 feet** in the best.

In conclusion, the tested hypothesis was determined to be valid in that it increased predictive accuracy over previously used methods. However the better predictive accuracy was provided by a datum we had not considered previously, namely the geometric center of the "cloud" of predicted shooter locations.

Many thanks to Mike Haag for his groundbreaking work on creating the concept of cones of uncertainty and providing an overall value to work with. I would also like to thank Leica Geosystems for the incredibly valuable ScanStation C10 that was used in this study and all of our case work in this area. Although I have not personally tested the alternative method of using strings or similar methods for trackback from trajectory rods, I am confident that the accuracy and resolution provided by the ScanStation is responsible for a large portion of the accuracy we were able to demonstrate in predicting the shooters location in this study and our casework. And finally, I am grateful to the thousands of scientists from history who conceived and implemented the idea of the "working model". At PSI our use of the 3D working model has consistently allowed us to achieve the type of accuracy and foundational validity that is required for forensic analysis.





.223 AR15 Rifle



.223 AR15 Rifle and Drywall Target



Car Door with Ballistic Trajectory Rods



Drywall with Ballistic Trajectory Rods



Leica Geosystems C10 ScanStation with Targets



Leica Geosystems ScanStation C10 Laser Scan of .45 Handgun Set-up

